

Comparative Effectiveness of Concrete and Virtual Manipulatives on Students' Learning of LCM and HCF and Classroom Engagement

Sagheer Ahmad*, Afshan Naseem** and Khush Naseeb***

Abstract

This study compares the effectiveness of concrete versus virtual manipulatives on fifth-grade students' mathematical achievement and classroom engagement when learning prime factorization, highest common factor (HCF), and least common multiple (LCM). A quasi-experimental pretest-posttest control group design was employed with 32 participants over a two-week period. Data were collected using mathematics achievement tests and the What is Happening in the Class? (WIHIC) engagement scale. One-way ANCOVA results revealed a statistically significant difference in posttest mathematics achievement between groups, $F(1,29)=13.289$, $p<.001$, $\eta^2=.314$, favoring the virtual manipulatives group ($M=13.88$, $SD=2.849$) over the concrete group ($M=10.88$, $SD=2.754$). Mixed-design ANOVA demonstrated significant interaction effects between time and group for both achievement, $F(1,30)=13.388$, $p<.001$, $\eta^2=.309$, and engagement, $F(1,30)=25.929$, $p<.001$, $\eta^2=.464$. Both manipulative types enhanced learning outcomes; however, virtual manipulatives yielded significantly greater improvements. These findings provide evidence-based guidance for educators and curriculum developers in selecting instructional tools to optimize mathematics learning and student participation.

Keywords: Concrete manipulatives, HCF, LCM, mathematical achievement, virtual manipulatives

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Introduction

In the contemporary era, technological innovation continues to accelerate and in turn, the use of virtual manipulatives in the classroom, especially in mathematics, is a vogue. This is so because a substantial number of students struggle to comprehend highly abstract and complex mathematical concepts in their elementary schooling (Khan et al., 2023; Ottenbreit-Leftwich & Kimmons, 2020). Manipulatives enable students to engage meaningfully with abstract mathematical concepts by facilitating visualization, strengthening conceptual understanding, nurturing mathematical reasoning, and supporting effective problem-solving (Moyer-Packenham & Bolyard, 2016). Although both concrete and virtual manipulatives contribute to mathematics learning, virtual manipulatives offer unique interactive features that can significantly enhance students' mathematical understanding and problem-solving skills (Leow & Kaur, 2024).

Conventionally, manipulatives have been in the form of concrete objects such as blocks, beads or cardboards factor tress that enable the students to physically manipulate and explore the mathematical relationships (Ramirez & Tabirao, 2023). However, the emergence of virtual manipulatives, that essentially replicate and extend the functions of their concrete counterparts, has introduced new openings for teaching and learning mathematics (Moyer-Packenham & Bolyard, 2016). Virtual manipulatives are dynamic in nature, having the potential to transform mathematical instruction by enabling students to manipulate, visualize, and experiment with mathematical concepts in ways that are not possible with static, concrete manipulatives (Bush, 2021; Kabel et al., 2021).

The elementary school fifth graders are at a crucial stage in their mathematical development. Fifth grade in elementary schooling is a pivotal year because students transition from concrete arithmetic to more abstract reasoning and build the fluency and problem-solving skills that underlie later mathematics success (National Council of Teachers of Mathematics [NCTM], 2014). Gaps from earlier grades can become more apparent, making targeted classroom supports for reasoning, justification, and mathematical communication important (National Research Council, 2001). At this very stage, it is essential for students to master basic concepts like Highest Common Factor (HCF) and Least Common Multiple (LCM) (Halim et al., 2017; Kurniasih et al., 2020). As these concepts provide the foundation for more complex topics including fractions, ratios and algebraic structures (Shoaib & Akhter, 2020; Halim et al., 2017; Kumar, 2020). However, it is generally perceived that the concepts like HCF and LCM are often abstract and difficult for the students to understand. This difficulty thereby leading to misconceptions in their future understanding of mathematics (Salamah & Sujadi, 2018; Yağmur, 2020; Triyani et al., 2012).

For instance, in a study involving 248 elementary students in Pakistan, Mohyuddin and Khalil (2016) found significant confusion between HCF and LCM learning, with only one-third of students answering correctly when asked to calculate the LCM of 12 and 18. Similarly, Shoaib and Akhtar (2020) identified misconception in HCF and LCM learning among 1500 Pakistani school students. These conceptual misunderstandings necessarily hinder students' ability to solve real-life problems in an efficient manner (Mohyuddin & Khalil, 2016). Thus, it seems necessary to search for effective instructional methodologies for teaching HCF and LCM so as to ensure the solid development of students' mathematical foundations.

In the very context of teaching complex mathematical concepts, the available literature reports diverse findings regarding comparative efficacy of virtual and concrete manipulatives (Foulkes et al., 2023; Muhammad et al., 2023; Shen, 2023). There are certain studies suggesting that the virtual manipulatives significantly enhance students' classroom engagement and their mathematical achievement resulting from their interactivity and visually appealing nature (Guan et al., 2024; Furner, 2024). These virtual manipulatives also contribute to differentiated instructional methodology, that permits the learners to perform corresponding to their cognitive processing rate and thereby engage in investigation and analysis of mathematical concepts in ways that suit their individual learning preferences (Moyer-Packenham et al., 2008).

Despite this, many studies indicate advantages of concrete manipulatives, particularly in improving mathematical achievement (Bornaa et al., 2023; Enuma & Ijeh, 2024; Jones & Tiller, 2017). Tangible experiences with concrete manipulatives are believed to offer students a more profound connection to the material, which can be crucial for developing a solid comprehension of complex mathematical concepts (Adesokan, 2024).

In prior studies, the researchers have extensively examined the comparative effectiveness of concrete versus virtual manipulatives across various mathematical concepts; this broader body of evidence remains markedly fragmented, with certain studies supporting the tactile immediacy of physical objects and others advocating for the interactive affordances of digital environments (Siller & Ahmad, 2024; Farra et al., 2024; Vessonen et al., 2021; Willingham, 2017; Stull & Hegarty, 2016; Moyer-Packenham et al., 2013; Carbonneau et al., 2013). Crucially, however, these studies have noticeably neglected the pedagogical application of either manipulative type to the specific concepts of Highest Common Factor (HCF) and Lowest Common Multiple (LCM). Given that instructional tools often interact uniquely with domain-specific content, the extant findings cannot be reliably generalized to this foundational area of number theory. Thus, the rationale for the current study is firmly grounded in the imperative to address this gap; its

primary purpose is to isolate and compare the distinct impacts of concrete and virtual manipulatives on both students' mathematical achievement and their levels of classroom engagement within the context of HCF and LCM instruction.

The findings of this study may contribute evidence-based insights: into how manipulatives can support the teaching and learning of abstract mathematical concepts like HCF and LCM, enabling teachers to adopt more effective instructional approaches that foster students' mathematical achievement and engagement in contexts such as Pakistan. The outcomes could also enhance our understanding of how the two types of manipulative affect students' achievements in HCF and LCM math as well as their engagement in the class.

Theoretical Perspectives

In particular, the study draws upon the constructivist learning theories perceived by Piaget (1952) and Vygotsky (1978). Piaget (1952) emphasized more the individual cognitive activities while Vygotsky (1978) stressed socio-cultural activities pertaining to learning (Brau, 2020). From the perspective of constructivism theory, manipulatives, whether concrete or virtual, are key in catalyzing cognitive development (Calibugan & Maghuyop, 2024). They allow students to engage with mathematical concepts to a greater extent and adopt experimentation principles in learning (Ondog & Kilag, 2024).

Piaget's cognitive development theory synergizes with the application of manipulatives in mathematics classroom, owing to the fact that Piaget theorizes that cognitive maturation operates in stages and that during the concrete operational stage, learners must be allowed to engage with concrete manipulatives (Lourenco, 2012; Ojose, 2008). This enables learners to transition from experiential learning to the application of theoretical reasoning, a key milestone in the learning of mathematics (Ondog & Kilag, 2024; McNeil & Uttal, 2009).

On contrary, sociocultural theory proposed by Vygotsky highlighting the significance of social dynamics and cultural objects in cognitive development of students (Mntunjani et al., 2018). From this perspective, manipulatives can be regarded as cultural tools that enhance mathematical achievement and classroom engagement of the students, through interactive environment where they can explore the relationships between mathematical concepts and the manipulatives (Lafay et al., 2019; Denhere et al., 2013).

The utilization of concrete and virtual learning tools (manipulatives) is seen as scaffolding agents in the context of this study. These manipulatives aid students in exploring and understanding concepts such as HCF and LCM. In the forthcoming sections of the study, researchers described the relevant literature regarding the efficacy of both

concrete and virtual manipulatives for facilitating elementary students mathematical achievement and classroom engagement.

Significance of LCM and HCF in Elementary Mathematics Learning

The mathematical concepts like HCF and LCM are fundamentals of number theory (Kumar & Lalima, 2024). In higher grades, we need these basic concepts for further entanglement of different mathematical concepts and their aligned reasoning. For a fifth grader, these concepts are more important than ever since it is the bridge from operations to abstract mathematical thinking (National Council of Teachers of Mathematics [NCTM], 2014).

The HCF defined as the largest numbers that two or more numbers can divide into each other without leaving a remainder, and LCM, least common multiple is also used to define the smallest one amongst two or more numbers, are not only just a tool to perform calculations but fundamental concepts that content specialists require teachers to be able to work on a fraction, ratio and proportion, and algebra (Fauzan et al., 2020). Research has demonstrated that when students develop a robust understanding of HCF and LCM, they are better equipped to find common denominators for fraction operations, solve real-world problems involving equal grouping and repetitive events, and build the number sense necessary for success in higher mathematics (Punjab Education Foundation, n.d.).

A substantial amount of research studies show that elementary students often encounter serious difficulties in learning elementary number theory. Among other things, they struggle with not fully understanding the relevant factors and multiples to challenges in applying these concepts to word problems and real-life situations (Mahmud et al., 2023, 2024). They identified three categories of learning obstacles ontological, epistemological, and didactic that impede elementary level students' understanding of LCM and GCF, noting that many students perceive these concepts merely as procedural factorization exercises rather than meaningful mathematical relationships. Due to this focus on procedures, students learn to memorize algorithms without grasping the underlying concepts, hence limiting their ability to transfer knowledge to new problem-solving situations (Foster, 2024).

Addressing Learning Challenges through Manipulative-based Scaffolding. In view of the documented challenges faced by students in comprehending the HCF and LCM concepts, it has been posited that the use of manipulatives as scaffolding supports in enhancing mathematical understanding of HCF and LCM concepts and classroom participation (Salsabela et al., 2025). Both concrete and virtual manipulatives help students create cognitive bridges making abstract concepts tangible. In other words, they make the relationship between factors and multiples visible. In turn, this establishes deep internal relationships (Ahmad & Siller, 2024; Carboneau et al., 2013; Moyer, 2001).

As per Kumar and Lalima (2024), suitable marbles of HCF and LCM concepts using individualized, manipulative-based instruction improve achievement of Grade V students. It is because the use of manipulatives caters to various learning styles and brings one's learning pace to each student. Using manipulatives while teaching HCF and LCM can be very useful as both of these concepts involve relationships. Also, being a relationship, either of factors or multiples is an abstract idea. In other words, they do not exist as physical objects but are rather relational properties of numbers. Students can develop mental images for these relationships and make them accessible by sorting out or physically constructing (or virtually constructing) objects, building factor trains with Cuisenaire rods, as well as visualizing through in-person representations (Clements & Sarama, 2014; Fyfe et al., 2014).

Furthermore, research on learning obstacles has emphasized the need for instructional designs that focus on conceptual understanding rather than procedural memorization, with manipulatives providing the concrete experiences necessary for students to move from enactive to iconic to symbolic representation of mathematical ideas (Mahmud et al., 2023). Through the use of manipulatives to discover common factors and multiples, students develop an understanding that is both procedural and conceptual. This understanding is important in using HCF and LCM in different mathematical contexts. It helps overcome epistemological obstacles that limit students' learning (Mahmud et al., 2024). The following two sections discuss concrete manipulatives and virtual manipulatives efficacy and influence on students' mathematical learning and engagement effect in classroom.

Effectiveness of Concrete Manipulatives

In mathematics education, manipulatives have long been considered an essential resource, especially for children in elementary school (Siller & Ahmad, 2024). Manipulatives provide learners with opportunities to engage with mathematical ideas through concrete, sensory experiences, allowing them to explore, represent, and construct meaning from abstract concepts (Ahmad & Siller, 2024). Research suggests that interacting physically with manipulatives can enhance students' learning experiences by supporting conceptual understanding and making mathematical relationships more accessible (Foulkes et al., 2023; Jones & Tiller, 2017). The use of manipulatives in math education has been shown to increase math-related educational goals. Students build the skills to understand manipulative relationships and strengthen that knowledge visually with the tools (Byrne et al., 2023; Enuma & Ijeh, 2024; Farra et al., 2024; Thomas et al., 2024). Moreover, the use of manipulatives active the attention of the students in learning and the process becomes more interactive with tools that allow for exploration (Warren, 2024; Kwon & Capraro, 2018).

Although the positive contributions of manipulatives have been thoroughly described in the literature, there are also some criticisms in the literature (Neesam, 2023; Foulkes et al., 2023). Surface level learning is a concern presented by McNeil and Jarvin (2007), where a student, in their opinion, becomes mesmerized with the concrete objects themselves, losing focus on the more meaningful learning of the fundamental mathematical concepts. Also, using real concrete manipulatives can take a long time, and can be hard for big groups or when working with complex math problems (Kwon & Capraro, 2018). That's why these limitations have led the researchers to investigate virtual manipulatives for use in math teaching.

The Rise of Virtual Manipulatives

In comparison to other manipulatives, digital manipulatives are easier to access, more flexible, and more interactive (Schäfer, 2021). Students can use the National Library for Virtual Manipulatives (NLVM) site to access interactive manipulatives and learning resources designed to support the teaching and learning of primary and secondary mathematics. These virtual environments allow students to explore mathematical concepts dynamically, including representations such as factor trees that support understanding of numerical relationships and operations (Moyer et al., 2002; Ahmad & Siller, 2024). Unlike concrete manipulatives, technology-based manipulatives can be customized to fit the learning needs of each student and adaptively facilitate engagement to varying degrees of learning and developmental challenges (Yakubova et al., 2023; Flevares et al., 2022).

Research has shown the effectiveness of virtual manipulatives, on the other hand, is demonstrated in the works of Justo et al. (2022) that virtual manipulatives enhance learners' experiences. Several research studies, not related to each other, specifically cite the increase in students' mathematical achievement due to the use of virtual manipulatives (Fisher, 2023; Keldgord & Ching, 2022; Rich, 2023; Silva et al., 2021; Vallejo-Vargas & Reid, 2024).

Manipulatives are not only locked to the increase of achievement but also the increase of motivation of the students. Students tend to use digital tools more than traditional concrete tools (Furner, 2024; Guan et al., 2024). Numerous studies have looked at the effects manipulatives have on different aspects of math. For example, Zhang et al. (2022) reported that elementary school students taught math using virtual manipulatives in a digital learning environment scored significantly higher than those taught using traditional methods. Moyer-Packenham et al. (2019) also reported that the use of manipulatives increased achievement and problem-solving skills in elementary math. In addition, Patterson et al. (2016) reported that virtual manipulatives improve student engagement, particularly in remote learning settings. Therefore, students are more positive

in a learning environment that utilizes manipulatives, and they achieve higher results at elementary level (Ahmad & Siller, 2024; Quane, 2025).

While there are benefits to using virtual manipulatives, some negative aspects also need to be considered. One of the major critiques of using technology in the classroom is the lack of physical interaction. Studies show toddlers and preschool-aged children and physical engagement with the manipulatives as it is especially necessary to build an understanding of abstract mathematical concepts (Keldgord & Ching, 2022; Vessonen et al., 2021). In addition, some researchers argued that the risk of cognitive overload might be amplified in the context of virtual manipulatives when excess information and distracting stimuli are present (Ibili, 2019; Moyer-Packenham et al., 2008). These varying conclusions highlighted the need for further studies to measure the effectiveness of virtual manipulatives in mathematics instruction compared to traditional concrete manipulatives when teaching HCF and LCM concepts to young students at elementary level.

Research Questions

Based on the literature discussed in theoretical perspective section, it was found out that there is not any particular study which deals with the effectiveness of concrete manipulatives and virtual manipulatives in teaching HCF and LCM. There is a multitude of studies that analyze the impact of these manipulatives on students' learning outcomes on a variety of mathematical concepts (Byrne et al., 2023; Enuma & Ijeh, 2024; Farra et al., 2024; Rich, 2023; Silva et al., 2021; Thomas et al., 2024; Vallejo-Vargas & Reid, 2024), but there is no such studies available in literature those specifically compare concrete and virtual manipulative assisted instruction in teaching HCF and LCM concepts to young students at elementary level. Keeping in view the significance of HCF and LCM concepts, a study seems to be indispensable to understand how both categories of manipulatives play their role in the pedagogical approaches of HCF and LCM. Thence fore, the study under consideration aims at addressing the literature gap explored by the research questions described here forth:

(RQ1): How do fifth grade students' mathematical achievement level, as measured by pre-post assessment scores differ when taught HCF and LCM using virtual manipulatives compared to concrete manipulatives?

(a1) How do the posttest scores of fifth-grade students differ when taught HCF and LCM using virtual manipulatives compared to concrete manipulatives?

(b1) Does the teaching with virtual manipulatives, compared to concrete manipulatives, impact mathematical achievement of fifth grade students over a period of intervention?

(RQ2): To what extent does the use of virtual manipulatives or concrete materials influence students' engagement level during HCF and LCM instruction in a fifth-grade mathematics classroom?

(a2) What is the comparative impact of virtual manipulatives and concrete materials on classroom engagement levels among fifth-grade students?

(b2) How do classroom engagement levels differ between fifth-grade students using virtual manipulatives and concrete materials over the intervention period?

Methodology

Design

This study employed a quantitative research approach, utilizing a true-experimental randomized pretest-posttest control group design to assess the effectiveness of virtual and concrete manipulatives integrated instruction on grade five students' mathematical achievement in the HCF and LCM concepts and their level of classroom engagement. Random assignment of participants to the instructional conditions was used to minimize selection effects and thereby strengthen the internal validity of the study and the credibility of casual inferences (Cohen et al., 2017). The study followed a pretest-intervention-posttest procedure. Prior to the intervention, students in both groups completed the same mathematics achievement test and the What Is Happening In Class (WIHIC) scale to establish baseline measures of mathematical achievement and classroom engagement. Following the intervention, the same measures were administered as posttests to determine changes in students' achievement and engagement. The two experimental conditions differed only in the type of manipulative-based instruction provided. One group received instruction supported by concrete manipulatives (concrete manipulative group), whereas the other received instruction supported by virtual manipulatives (virtual manipulative group). This experimental arrangement enabled a direct comparison of the two instructional approaches while controlling, to the extent possible, for pre-existing differences between participants. Consequently, differences in posttest outcomes between the groups could be examined as evidence of the relative effectiveness of concrete and virtual manipulative integration in teaching HCF and LCM.

Sampling

The sample comprised 32 grade five students at government elementary school which were selected by means of convenience sampling technique, based on accessibility, willingness to participate, and feasibility within the study timeline (Thomas, 1997). After selection, random assignment of these 32 students to the two conditions resulted in a control group, that is, concrete manipulative group ($n = 16$) and an experimental group, that is, virtual

manipulative group ($n = 16$). The randomization ensured that the groups had a probability similar on baseline characteristics, mitigating potential confounding variables prior to the intervention. In order to ensure this balance, pretest scores for the Mathematics Achievement Test on concepts of HCF and LCM and the What Is Happening In Class (WIHIC) engagement scale were analyzed statistically. As presented in Table 1, the Mann-Whitney U test revealed no statistically significant difference in mathematical achievement between the virtual manipulative group ($Md = 5.00$, $n = 16$) and the concrete manipulative group ($Md = 4.50$, $n = 16$), $U = 127.00$, $z = -0.038$, $p = 0.97$, with a negligible effect size of $r = 0.007$. Similarly, Table 2 shows that an independent samples t-test indicated no significant difference in classroom engagement scores between the virtual manipulative group ($M = 3.05$, $SD = 0.14$) and the concrete manipulative group ($M = 3.12$, $SD = 0.19$), $t(30) = 1.24$, $p = 0.29$, two-tailed, with a very small effect size (Cohen's $d = 0.05$) and a 95% confidence interval ranging from -0.05 to 0.19 . The results indicated that both groups were statistically equivalent at baseline; therefore, a strong foundation was established for the subsequent comparison.

Table 1

Comparison of Pretest Scores Between both Groups in Mathematical Achievement

Groups	N	M	Median	SD	U	z	p	r
Concrete Manipulative Group	16	4.94	4.50	1.61				
Virtual Manipulative Group	16	5.00	5.00	2.31	127.00	-0.038	0.97	0.007

As indicated by the Mann-Whitney U test results in Table 1, no statistically significant difference was observed in mathematical achievement (HCF and LCM achievement test) scores between virtual manipulative group ($Md = 5.00$, $n = 16$) and concrete manipulative group students ($Md = 4.50$, $n = 16$), with minimal effect size of $r = 0.007$. Consequently, pre-assessment scores (baseline data) for both groups demonstrated equivalence before the implementation of the intervention.

Table 2

Comparison of Pre-assessment Scores for both Groups on the WIHIC Engagement Scale

Groups	N	Mean	SD	df	t-value	Sig.	Cohen's d
Concrete Manipulative Group	16	3.12	0.195				
Virtual Manipulative Group				30	1.24	0.28	0.05

Virtual Manipulative Group	16	3.05	0.139
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Table 2 indicates the results of an independent sample t-test and reveals that the pre-test scores of virtual manipulative group students were not significantly different ($M = 3.05$, $SD = 0.139$) compared to the students of concrete manipulative group ($M = 3.12$, $SD = 0.19$; $t(30) = 1.24$, $p = 0.288$, two-tailed). The mean difference in classroom engagement scale (WIHIC) scores was $= 0.07$, with a 95% confidence interval ranging from -0.05 to 0.19 . The Cohen's d statistic (0.05) indicated a very small effect size. Which means the pretest scores for both groups demonstrated equivalence before the implementation of the intervention.

Instrumentation

Two assessment tools, namely a mathematical achievement test for HCF and LCM; and the What Is Happening In Classroom (WIHIC) scale for measuring classroom engagement, were employed to address the research questions. The mathematics achievement tests (pretest and posttest) comprised 20 multiple-choice items developed from the grade five mathematics textbook chapter on LCM and HCF, designed to measure students' mathematical achievement at both the initial and final stages of the intervention.

A pilot study with 100 students from a comparable population was conducted in summer 2024 for the establishment of the validity and reliability of the instruments. Two context experts from the content field of mathematic examined item's validity, evaluated items clarity and wording, and also ensured each item was fully aligned with the fifth-grade curriculum's objectives. In the meantime, two instrument development specialists ensured the methodological rigor of construct validity of the mathematics achievement (HCF & LCM concepts) test. Following expert advice, items assessed as ambiguous or misaligned were modified or deleted, and Content Validity Index (CVI) was calculated. The scale-level CVI was 0.85 , greater than the acceptable minimum of 0.80 (Polit & Beck, 2006) thereby confirming the mathematics achievement test content validity.

After completing pilot study, item analysis was performed to further refine this mathematics achievement test. Items with discrimination index over 0.20 and difficulty index with the range 0.30 to 0.70 were retained (Munir et al., 2013). The final test confirmed an average (mean) item discrimination index of 0.41 and an average (mean) difficulty index of 0.39 . This means that the test items were able to differentiate good students from poor students well enough and the level of difficulty was acceptable enough. Results of item analysis are illustrated in Table 3. The reliability of the achievement test was estimated using the Kuder-Richardson Formula 20 (KR-20). The KR-20 test produced a coefficient of 0.80 which is above the acceptable coefficient of 0.70 (Nunnally &

Bernstein, 1994). This means that the final test has a good internal consistency to measure students' achievement.

These comprehensive validity and reliability procedures ensured that mathematical achievement for HCF and LCM instrument was psychometrically sound and appropriate for measuring the intended outcomes in this intervention study.

Table 3

Mean Item Discrimination and Difficulty Indices of the Mathematics Achievement Test

Source	N	Average Item Discrimination	Average Item Difficulty
Mathematics Achievement Test for HCF and LCM	20	0.41	0.39

A system was devised for reviewing the results of the mathematics achievement test for HCF and LCM. A score of '1' was given for correct calculations and results, while scores of '0' were given for results that were incorrect and for answers that did not show any work. The average score for the math section of the achievement test was 13, and the standard deviation was 3.01. This specific information is presented in Table 4. Along with that Table 5 provide examples of mathematics achievement test items for HCF and LCM.

Table 4

Descriptive Analysis of Piloted-Test for HCF and LCM

Source	N	Mean	SD
Mathematics Achievement Test for HCF and LCM	100	13	3.01

Table 5

Example Items from Mathematics Achievement Test for HCF and LCM

Concepts	Number of Items	Example Items
Least Common Factor (LCM)	5 (MCQs)	1. What is the LCM of 12 and 18 using prime factorization? a. 24 b. 36 c. 72 d. 108
Highest Common Factor (HCF)	5 (MCQs)	2. The HCF of two or more than two numbers, which have no common prime factor is ...? a. 1000 b. 100 c. 10 d. 10000

In its essence, the second instrument utilized in this study was What Is Happening In this Class (WIHIC), developed by Fraser and colleagues (1996). It is currently the most frequently used classroom environment instrument worldwide (Skordi & Fraser, 2019). This instrument examines the psychosocial climate of the classroom through students' perceptions of their learning environment, providing valuable insights into elements that can either foster or impede effective learning (Fraser, 2023).

The WIHIC comprises 56 statements employing a 5-point Likert scale (1 = Almost Never to 5 = Very Often). The instrument is organized into seven scales, each containing eight items: (i) Student Cohesiveness, (ii) Teacher Support, (iii) Involvement, (iv) Investigation, (v) Task Orientation, (vi) Cooperation, and (vii) Equity (Fraser, 2023; Skordi & Fraser, 2019). These scales are represented and exemplified in Table 6.

In addition, the reliability coefficient using Cronbach's Alpha was calculated for the whole WIHIC scale. Descriptive statistics and reliability coefficients for the overall scale and its sub-dimensions are detailed in Table 7. Prior to data collection, the scale underwent careful piloting to ensure its suitability for assessing pre- and post-intervention phases.

Table 6
Example Items for What Is Happening in Classroom (WIHIC) Scale

Factors/Constructs	No. Of Items	Example Items
Student Cohesiveness	8	I make friendships among students in this class. I work well with other class members.
Teacher Support	8	The teachers take a personal interest in me. The teachers talk with me.
Involvement	8	I discuss ideas in class. I am asked to explain how I solve problems.
Investigation	8	I carry out investigations to test my ideas. I solve problems by using information obtained from my own investigations.
Task Orientation	8	I do as much as I set out to do. I pay attention during mathematics class.
Cooperation	8	I cooperate with other students when doing assignment work. I learn from other students in this class.
Equity	8	I get the same amount of help from the teacher as do other students.

My work receives as much praise as other students’ work.

The reliability of the WIHIC was assessed using Cronbach’s Alpha coefficient, which demonstrated acceptable internal consistency for the overall scale ($\alpha = 0.71$). Descriptive statistics and reliability coefficients for the overall scale are detailed in Table 7. Prior to data collection, the scale underwent careful piloting to ensure its suitability for assessing both pre- and post-intervention phases.

Table 7
Descriptive Statistics and Reliability Coefficient for WIHIC Scale

Scale	No. of Items	Mean	SD	Cronbach Alpha
WIHIC Scale	56	180.54	13.36	0.71

The WIHIC has demonstrated robust validity and reliability across diverse educational contexts and has been translated into multiple languages including Chinese, Greek, Spanish, Indonesian, Korean, and IsiZulu (Skordi & Fraser, 2019). Factor analytic studies consistently support the instrument’s seven-scale structure, with previous research reporting percentage of variance accounted for ranging from 3.19% to 28.71% for different scales, and total variance explained typically exceeding 64% (Fraser, 2023; Skordi & Fraser, 2019).

Procedure of the Study

Participants were selected after a formal approval of SREC of the institute. Among 32 fifth grade students, 16 were assigned randomly to the virtual manipulative group, while the other 16 students were assigned to the concrete manipulative group. Both the groups were undergone for intervention over the period of 2 weeks. The concrete manipulative group, just like the virtual manipulative group, received four lessons a week, each lasting 90 minutes, amounting to 360 minutes of instruction. It is worth mentioning that the same teacher taught both groups in order to control teacher-related variables.

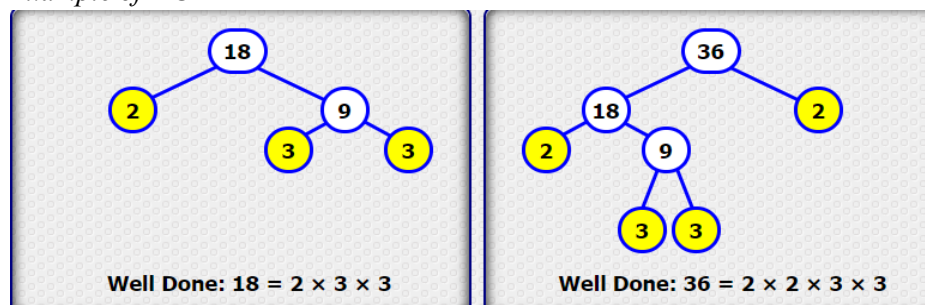
Prior to the intervention, all students in both groups completed pre-tests, which included a mathematics achievement test assessing concepts of Highest Common Factor (HCF) and Lowest Common Multiple (LCM), alongside the What Is Happening In this Class (WIHIC) scale to measure perceptions of the classroom learning environment. The intervention followed the same timeline for both groups spanning 14 days with 360 minutes of instruction divided into four sessions per week. After the intervention period, all participants in both groups completed post-tests (same as pretest) comprising mathematics achievement test and the WIHIC scale to find out any change in their academic performance and perception about learning environment.

Instructional Settings

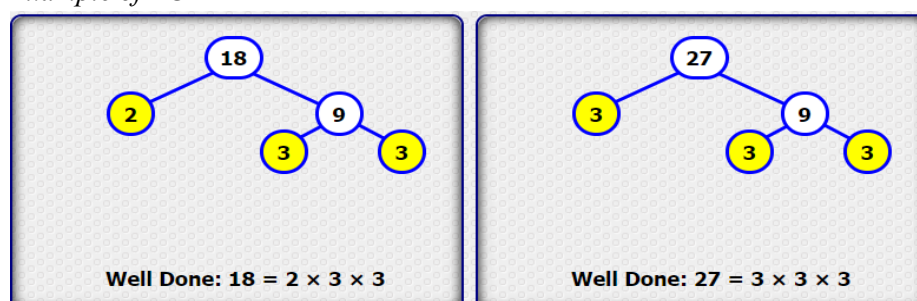
The study was conducted within the computer lab equipped with 50 individual workstations, ensuring each student had access to their own computer for independent work. For two weeks of the intervention phase the virtual manipulative group was instructed with the help of virtual manipulatives. Specifically, Transum Factor Tree Application was used by the class teacher to teach the concepts of HCF and LCM to fifth graders. The Transum (Transum.org) was selected for this study because it offers a free, accessible, and pedagogically sound platform for creating virtual factor trees. Class teacher utilized the dedicated “Factor Trees” activity purposely designed to enable students to visualize the prime factorization of number. Transum is a browser-based interactive tool requiring no student login to access the core learning content. Students type whole numbers greater than one into the circles of a factor tree, ensuring each branch ends in a prime number.

This activity has a ‘check’ button which provides feedback and allows the students to verify their problem-solving steps before moving to the next steps. Multiple difficulty levels, Challenge 1 to 4 and Advanced offer differentiated instruction. They can also use an HCF and LCM calculator to verify or demonstrate. While teacher login is required to access answer keys and the Class Admin system for progress tracking, these premium features were not necessary for student participation in this intervention. Research consistently demonstrates that virtual manipulatives offering dynamic representation and immediate feedback enhance conceptual understanding more effectively than concrete manipulatives, making Transum Factor Trees a pedagogically sound choice for teaching prime factorization, HCF, and LCM.

During the intervention for virtual manipulative group the lesson begins with a 15-minutes introduction by the teacher, who explained the concepts of factors, multiples, HCF and LCM. The introduction set the stage for the use of Transum. Following the introduction, a 15-minute demonstration was provided. Then teacher projected the Transum on a smartboard and demonstrated how to use the virtual factor trees tool to break down numbers into their prime factors. An example was used for finding the HCF of 18 and 36 on the first day of intervention and subsequent to this all the problems from basic to advanced level in the fifth-grade national curriculum related to HCF were taught with the same process throughout the first week of intervention. Figure 1 represents the students working with HCF problem with the help of virtual manipulatives.

Figure 1*Example of HCF*

In addition, in the second week of the intervention on the first day an example was used for finding the LCM of 18 and 27 and subsequent to this all the problems from basic to advanced level in the fifth-grade curriculum related to LCM were taught with the same process throughout the second week of the intervention or till the end of the intervention. Each step was shown in real time, and the interactive nature of the software allowed for immediate visualization of prime factors and their relationships. Figure 2 depicts the students working with LCM problems with the help of factor tree virtual manipulatives.

Figure 2*Example of LCM*

In the subsequent 20 minutes guided practice session, students were provided with computers equipped with the Transum. They were divided into the pairs and given different questions to determine HCF in the first week and LCM in the second week to work on. The teacher circulated the classroom as students started their virtual factor tree exercises. Students felt encouraged to participate while learning together.

After twenty minutes of silent work on making virtual factor trees, students started to find the HCF and LCM of other numbers. They were each given sheets of paper to record

their answers and to discuss the results with their peers to identify. Students were able to identify any discrepancies, and this strengthened the students' understanding of the topics. At the end, lesson was concluded with a 20-minute review session where the teacher summarized the key points and invited a few students to share their screens and present their virtual factor trees. Students explained their process of finding the HCF and LCM, which provided an opportunity for reinforcement of the material.

In the concrete manipulative group teacher utilized “cardboard factor trees” to teach the concepts of HCF and LCM to fifth graders. The lesson begins with a 15-minutes introduction by the teacher, who explained the concepts of factors, multiples, HCF, and LCM. Then teacher emphasized the importance of these concepts in various mathematical applications. This introduction served as a foundation for the subsequent hands-on activity using cardboard factor trees. Following the demonstration, a 15-minute demonstration was conducted. The teacher used large cardboard cutouts to create factor trees for numbers such as 23 and 46 for finding the HCF on the first day of intervention and subsequent to this all the problems from basic to advance level in the fifth-grade national curriculum related to HCF were taught with the same process throughout the first week of intervention. In addition, in the second week of the intervention on the first day an example was used for finding the LCM of 23 and 46 and subsequent to this all the problems from basic to advanced level in the fifth-grade curriculum related to LCM were taught with the same process throughout the second week of the intervention or till the end of the intervention. Each step of the factorization process was demonstrated, showing how to break down the numbers into their prime factors using the cardboard pieces. This visual and tactile representation helped students grasp the concept of prime factorization. Figure 3 shows the students working with cardboard concrete manipulatives in control groups.

Figure 3
Students Working with Cardboard



In the 20-minute guided practice session, students were provided smaller cardboard pieces to create their own factor trees in pairs. They were divided into the pairs and given different questions to determine HCF in the first week and LCM in the second week to work on. The teacher was walking around the classroom helping all the pairs make their factor trees. This made it easy for the students to help each other and get even more familiar with the lesson. In the twenty minutes they all had to themselves, everyone made individual factor trees for a new problem set to find the HCF and LCM. They completed the trees and compared their answers with one another to talk about the differences and double-check their calculations. This made them get even more practice with the ideas to make them more confident with their knowledge.

At the end, lesson concluded with a 20-minute review session where the teacher summarized the key points and invited a few students to present their cardboard factor trees to the class. Students explained their process of finding the HCF and LCM, which provided an opportunity for reinforcement of the material. Table 8 offers an extensive and detailed breakdown of sample concrete and virtual manipulatives employed in teaching both the virtual manipulative group and concrete manipulative group.

Table 8

Selection of Physical and Digital Manipulatives

Concepts	Concrete Manipulatives	Virtual Manipulatives
Least Common Multiple (LCM) & Highest Common Factor (HCF)	Factor Trees with the Help of Card boards	Factor Trees with Digital Tool (Transum Mathematics) https://transum.org/Software/

Data Analysis

Data analysis was conducted using SPSS 28.0. The selection of parametric tests hinged upon results of the normality assessment. Following statistical approaches were adopted to respond the research questions.

To examine the effect of integrating manipulatives in teaching, treated as an independent variable, on the mathematical performance of grade five students in HCF and LCM concepts, a one-way ANCOVA test was conducted to evaluate statistical differences between the non-experimental and treatment groups after the intervention (T2), to investigate research questions (a1) and (a2). This approach, as suggested by Pallant (2020), is appropriate for analyzing interventions in a two-group pre/post-test design, particularly in cases where random assignment of participants is not feasible. To begin this intervention,

pre-assessment scores were taken so that participants' differences could be controlled, and systematic bias could be reduced, but this method does not get rid of the differences. Before conducting this test, there were a number of assumptions, such as data quality checks that included missing data, normality of data distribution, linearity, and outlier detection, as well as multicollinearity, singularity, homogeneity of variances, homogeneity of regression, and reliabilities of the covariates.

Additionally, a mixed-design ANOVA, incorporating both between- and within-subject factors (Split-Plot ANOVA/SPANOVA), was employed to examine data gathered at two separate time points (T1 and T2) to assess the progress made by groups over the intervention period, addressing research questions (b1) and (b2). This method focuses on finding main effects in time (pre & posttest), impact of the control condition (treatment effect), and possible interaction effect of the variables (influence of the independent variable, in this case, control condition effect, on the dependent variable, outcome, in relation to the moderator variable). The study presents the results of this process (measurement scale used in study, together with a test of homogeneity of variance) (Levene's test) and equality of covariance (Box's test for equality of covariance matrices), to the fullest extent possible for underlying assumptions. In depth consideration of the assumptions underlying the validity of the study, and the application of statistical analysis to study and data, results in the study's conclusion being reliable and valid.

Results

Table 9 shows how concrete versus virtual manipulatives assisted instruction impact students' math scores in the two groups. It shows the differences in average scores and standard deviation before and after the tests. Also, Tables 11 and 12 provide careful scrutiny of the raw data and present the findings of one-way ANCOVA and mixed between and within subjects ANOVA, which explain the significance of the changes in mathematics achievement test scores for both concrete manipulative group and virtual manipulative group. Without and with the changes, these findings are useful in offering more insight on the impact of the intervention and respond to the research question 1 (a1 and b1).

Table 9

Descriptive Statistics for Pre- and Post Mathematical Achievement Tests of both Groups

Assessment Phase	Concrete Manipulative Group	Virtual Manipulative Group
Pre-assessment		
<i>M</i>	4.94	5.00
<i>SD</i>	1.611	2.309
<i>N</i>	16	16
Post-assessment		
<i>M</i>	10.88	13.88

<i>SD</i>	2.754	2.849
<i>N</i>	16	16

Table 9 has both groups mean mathematics achievement test scores in HCF and LCM concepts and sheds light on the influence of the type of instruction students achievement. The concrete manipulative group had an increase in mean mathematics achievement test scores from M (pre) = 4.94 to M (post) = 10.88. Which means the post-test scores has been improved as compared to pre-test. As $SD = 1.611$ for the pretest, $SD = 2.754$ for posttest tells us how the scores were spread out for each of the phases. Likewise, the virtual manipulative group also had an increase in the mean mathematics achievement test from M (pre) = 5.00 to M (post) = 13.88. Consequently, which shows that post-test scores of this group have been improved as compared to their pre-test scores. The standard deviation (SD) = 2.309 for pretest, $SD = 2.849$ for posttest tells us how the scores were spread out for the pre- and post-tests of the two phases.

Additionally, the classroom engagement level of both groups was compared, mean scores and standard deviations for both groups in their pre- and post-tests are presented in Tables 10, 13, 14 showing the outcomes of one-way ANCOVA and mixed-design ANOVA demonstrating varied outcomes which also addresses the part of the research question 2 (a2 and b2).

Table 10

Descriptive Statistics of Pre-Posttest Scores for both Groups in WIHIC Scale

Assessment Phase	Concrete Manipulative Group	Virtual Manipulative Group
Pre-assessment		
<i>M</i>	3.125	3.050
<i>SD</i>	0.195	0.139
<i>N</i>	16	16
Post-assessment		
<i>M</i>	3.803	4.134
<i>SD</i>	0.253	0.113
<i>N</i>	16	16

Table 10 demonstrates the descriptive statistical results in pre- and post-test classroom engagement scale difference for both groups. For the concrete manipulative group, mean value for classroom engagement scale in post-test increase M (post) = 3.80 as compared to their mean value in pre-test M (pre) = 3.12. Which shows that the level of their classroom engagement has increased after participation in intervention. As, standard deviations (SD) = 0.195 for pretest and $SD = 0.253$ for posttest for concrete manipulative group illustrated how different in value the scores were per each phase of test. Likewise, for the virtual manipulative group mean values also increased in posttest as compared to

pretest. The scores increased and it went from $M(\text{pre}) = 3.05$ to $M(\text{post}) = 4.13$. The same kind of standard deviations ($SD = 0.139$ for pretest, $SD = 0.113$ for posttest also showed how different in value for each phase of test administration. In conclusion, both kind of concrete and virtual manipulative assisted instruction increased the students' classroom engagement level to some extent.

Research Question (a1)

To assess the effect of teaching HCF and LCM concepts using virtual manipulatives compared to concrete manipulatives on the mathematics achievement posttest scores, a one-way ANCOVA test was performed addressing research question 1 (a1). Results are presented in Table 10 in the context of descriptive statistics, whereas Tables 11 and 12 presenting a structured comparison or change in scores between both groups performance in pre- and post-tests (mathematics achievement test and WIHIC scale).

Table 11

One-Way ANCOVA Analysis of Post-assessment Results for both Groups

Source	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>p</i>	η^2
Intercept	1	281.724	281.724	53.880	<.001	0.650
Covariate (Pretest Achievement)	1	83.868	83.868	16.040	0.356	0.012
Groups (Concrete Manipulative/Virtual Manipulative)	1	69.485	69.485	13.289	0.001	0.314
Error	29	151.632	151.632			
Total	32	5208.000	5208.000			

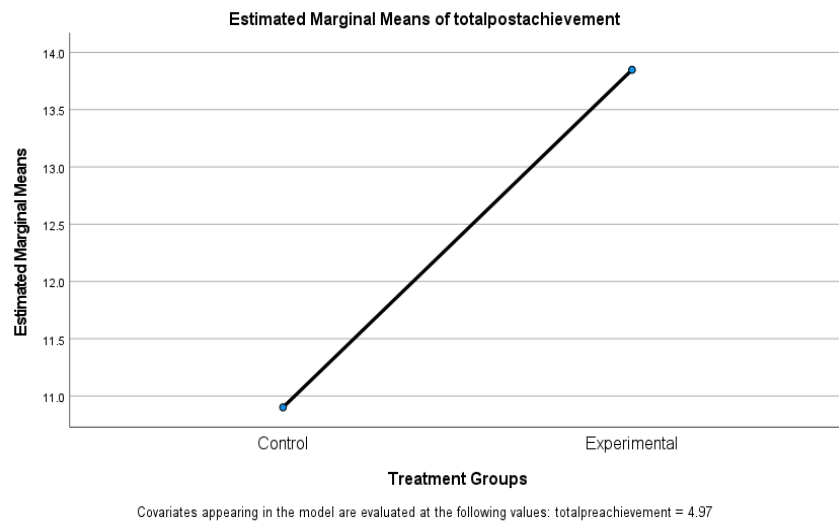
Table 11 shows the outcomes of one-way of ANCOVA and the effect of two interventions of fifth graders' mathematical achievement in HCF and LCM. The independent variable of the study is the kind of intervention used, i.e., virtual and concrete manipulatives assisted teaching, whereas the dependent variable is the mathematical achievement posttest scores.

Scores in the covariate pre intervention test were used to further enhance the analysis, and a preliminary check, in this case, validated the assumptions of the analysis of variance. After controlling pretest scores, there was a statistically significant difference between the concrete manipulative group and virtual manipulative group posttest mathematics achievement test scores $F(1, 29) = 13.289$, $p < 0.001$, and the effect size was moderate (partial eta squared = .314). The adjusted mean score of the virtual manipulative group was higher than that of the concrete manipulative group.

Weak positive correlation of pre and posttest performance (partial = 0.012) indicates that pretest scores did not contribute much, and thus there was little intervention difference on post scores. Virtual manipulative group ($M = 13.88$, $SD = 2.849$) also had a significant difference of post test scores compared to concrete manipulative group ($M = 10.88$, $SD = 2.754$), as shown in Table 9 and Figure 4.

Figure 4

Comparative Analysis of Posttest Results between both the Group



Research Question (b1)

A mixed-design ANOVA analysis was performed to answer b1 research question. Specifically, the focus was on measuring the mean scores pre and post assessments for both groups during the intervention period. The researchers focused on that part of Table 11 regarding mixed-design variations to evaluate the differences in performance for each of the groups during the intervention.

Table 12

Analysis of Variance for both Groups across Pre- Post-assessments

Source	df	SS	MS	F	p	Eta Sq.
Between Subjects						
Treatment Groups	1	37.516	37.516	4.062	0.05	.119
Error 1	30	277.094	9.236			
Within Subjects						
Pre-Post	1	877.641	877.641	340.418	<.001	.919

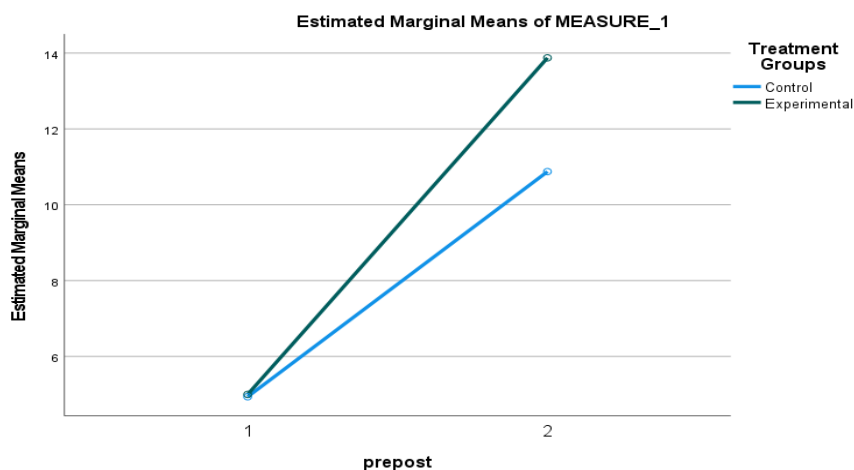
Pre-Post*Groups	1	34.516	34.516	13.388	<.001	.309
Error 2	30	77.344	2.578			

A mixed-design ANOVA was conducted with time as the within-subjects factor and instructional group as the between-subjects factor. The results showed a significant main effect of time, $F(1, 30) = 13.388$, $p < 0.001$, partial eta squared = 0.309, Wilks' Lambda = 0.691, indicating that students' scores increased from pretest to posttest. Marginally more than 30 subjects showed positive differences in performance on the mathematics achievement test at both levels of measurement: pre-intervention and post-intervention, evidenced by a main effect Wilks' Lambda = 0.081, $F(1, 30) = 340.418$, $p < 0.001$, partial eta squared = 0.919.

There was also a difference in the effectiveness of the two types of teaching approaches (concrete and virtual manipulative assisted teaching) by $F(1, 30) = 4.062$, $p = 0.05$, and the partially squared eta was 0.119. The justifiable explanation of the results of the assessment was analyzed in Table 11, and a positive explanation of the results was demonstrated in Figure 5, which was separated by periods of the test and was placed on the test performance of the students, thus identifying a high difference in the level of attainment of the student in the test achievement sample. The results confirmed the positive difference in the scores obtained in the posttest performed within the virtual manipulative group relative to the scores obtained in the pretest, and the relative measure of the difference in the scores obtained in the pretest and posttest performed on descriptive sample of test 9. The results provided important information on the use of virtual manipulative teaching to improve achievement in learning math and for other supportive methods to improve school math educational achievement.

Figure 5

Treatment Groups (Mathematics Achievement Pre-Posttest Comparison)



Research Question (a2)

To investigate the impact of teaching HCF and LCM using virtual manipulatives versus concrete manipulatives on classroom engagement level posttest scores, a one-way ANCOVA test was implemented (research question 2(a)). Results are presented in Table 10 for descriptive statistics, with Tables 13 and 14 offering a comparative summary of the findings.

Table 13

ANCOVA Investigating WIHIC Post-assessment Scores for both Groups

Source	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>p</i>	η^2
Intercept	1	0.860	0.860	22.980	<.001	0.442
Covariate (WIHIC Pretest)	1	0.072	0.072	1.915	0.177	0.062
Groups (Control/Experimental)	1	0.945	0.945	25.269	<.001	0.466
Error	29	1.085	0.037			
Total	32	506.064				

Table 13 reveals the results of the one-way ANCOVA analysis of classroom engagement level of the two interventions for the fifth graders in the classroom. After controlling for pre-test scores of WIHIC scale, there was a statistically significant change to be seen between the concrete manipulative and the virtual manipulative group in the post WIHIC scale results ($F(1, 29) = 25.269$, $p < 0.001$) with a medium effect (partial eta squared = 0.466). This result indicates that students in the virtual manipulative group reported higher classroom engagement than those in the concrete manipulative group after the intervention.

Moreover, there was some interaction with the pretest and posttest WIHIC scores (partial eta squared = 0.062) suggesting the pretest scores minimally affect the posttest engagement scores. Importantly, the virtual manipulative group ($M = 4.134$, $SD = 0.113$) had statistically significant posttest scores in contrast to the concrete manipulative group ($M = 3.803$, $SD = 0.253$) because they had virtual manipulatives during the instruction.

Research Question (b2)

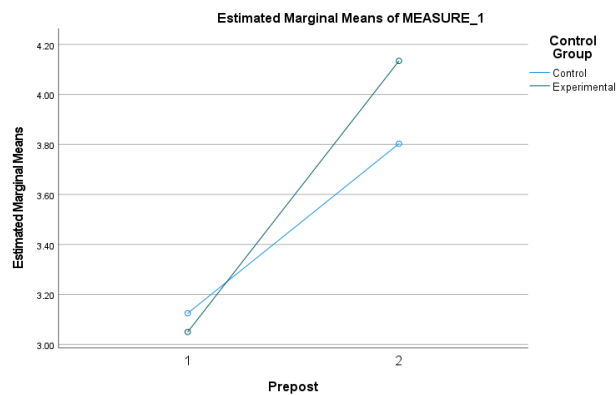
A mixed-design ANOVA was conducted to examine both within- and between-subject effects in order to address b2 research question. The primary aim was to assess the differences in mean WIHIC scores between pre- and post-assessment for both the groups during the treatment period. Researchers focused on Table 14's section detailing within-subject differences to analyze differences in classroom engagement levels within each group throughout the intervention duration.

Table 14*Analysis of Variance for both Groups across Pre- Post-assessments in WIHIC*

Source	df	SS	MS	F	p	Eta Sq.
Between Subjects						
Treatment Groups	1	0.263	0.263	6.249	0.01	0.172
Error 1	30	1.260	0.042			
Within Subjects						
Pre-Post	1	12.423	12.423	488.936	<.001	0.942
Pre-Post*Groups	1	0.659	0.659	25.929	<.001	0.464
Error 2	30	0.762	0.025			

A mixed-design ANOVA was conducted to examine both within- and between-subject effects demonstrate significant differences between treatment groups (concrete manipulative group vs. virtual manipulative group) and measurement phases (pretest and the posttest), with Wilks' Lambda = 0.536, $F(1, 30) = 25.929$, $p < 0.001$, and partial eta squared = 0.464. Each group students showed a major increase in their WIHIC test results post intervention versus pre intervention phases, $p < 0.001$. Wilks' Lambda = 0.058, $F(1, 30) = 488.936$ showing increase in test results were a function of the intervention versus random chance, also and statistically significant main effect Wilks' Lambda.

Moreover, the difference in the type of teaching (concrete vs. virtual manipulative teaching) was also statistically significant, $F(1, 30) = 6.249$, $p = 0.01$. Table 13 shows information about it. Figure 6 also backs up the results showing pre intervention vs post intervention WIHIC test score intervention score gap and the positive test score increase contribution due to the intervention. Virtual manipulative group also showed more than average gains between the post and pretest score results (see Table 10). These results highlight the type of teaching strategies, especially with the use of virtual manipulatives, encourage students to participate more during the lesson.

Figure 6*Treatment Groups (WIHIC Pre-Posttest Comparison)*

Discussion

To determine the effect of manipulatives (concrete and virtual) across different environments on grade five learners' HCF and LCM mathematical achievements and classroom engagement, this research set out to achieve the above stated objectives. The significance of investigating these particular mathematical concepts lies in their foundational role within elementary number theory, as they represent the critical juncture where students transition from concrete arithmetic operations to the abstract relational thinking required for fractions, ratios, proportions, and eventual algebraic reasoning. Given that the literature identifies persistent ontological, epistemological, and didactic obstacles that reduce these concepts to mere procedural factorization in students' minds, the present findings offer empirical evidence on how different scaffolding tools can either reinforce or dismantle these barriers, thereby shaping long-term mathematical trajectories.

Difference between Mathematical Achievement (RQ-1)

A part of this study focused on comparing HCF and LCM math skills between those receiving virtual manipulatives to those with concrete manipulatives with regards to achievement outcomes. The outcomes were positive, and the virtual manipulatives-based tailored interventions successfully enhanced the mathematics achievement (RQ a1). Virtual manipulative group students posttest scores ($M = 10.88$, $n = 16$) were higher than comparative to the scores of concrete manipulative group students ($M = 13.88$, $n = 16$). There is a medium level of effect between the two groups (partial eta square = 0.314). This differential gain carries substantial pedagogical weight, because mastery of HCF and LCM is not an isolated endpoint but a prerequisite for fraction operations and proportional reasoning in later grades.

The virtual manipulatives, by offering dynamic and customizable representations such as interactive factor trees, appear to have provided a more effective cognitive bridge that allowed students to move beyond rote memorization of algorithms toward a genuine comprehension of numerical relationships, a transition that cognitive development theory posits as essential during the concrete operational stage. Findings of prior experimental studies in mathematics, for example Zhang et al. (2022), are consistent with the conclusion of this study. Studies from various domains of mathematics (Foulkes et al., 2023; Muhammad et al., 2023; Shen, 2023) show a comprehensive analysis which suggests that usage of virtual manipulatives has a positive effect on the mathematical performance of students at elementary level. In conjunction with the studies mentioned, meta-analytic outcomes on the advantages of utilizing virtual manipulatives in teaching of mathematics (Moyer-Packenham et al., 2019; Patterson et al., 2016) correspond with the empirical evidence generated in the current research. The medium effect size observed here is particularly noteworthy for elementary number theory, as it suggests that the scaffolding

provided by virtual tools does not merely produce superficial test improvements but actively restructures students' understanding of factor and multiple relationships, thereby addressing the epistemological obstacles that typically limit transfer to novel problem-solving contexts.

Additionally, a mixed-design ANOVA between-within-subject was conducted to assess variations in pre- and post-assessment scores concerning math performance to answer the research question b1. In terms of mathematical achievement in HCF and LCM, the findings revealed a statistically significant increase in mean scores between the pretest ($M = 5.00$, $SD = 2.309$) and posttest ($M = 13.88$, $SD = 2.849$) of virtual manipulative group students. A significant main effect was observed for pre- and post-assessment scores, Wilks' Lambda = 0.081, $F(1, 30) = 340.418$, $p = 0.00$, partial eta squared = 0.919. Both groups showed improved in math performance during both the assessment phases. Statistical results of partial eta squared show a large effect size. After implementation of the intervention, students' achievement in mathematics of virtual manipulative group was significantly improved over pre-implementation. The scale improvement indicates that this student who had a near floor performance at pretest was able to perform nearly 14 out of the possible scale. For primary teachers the data shows that giving students the chance to build mental models using interactive virtual representations does not lead to students memorizing the experimental procedure only but rather they were internalizing properties of numbers. This will assist the development of number sense and getting ready for the multiplicative reasoning that middle school math demands.

However, the findings of study conducted by Zhang et al. (2022) confirms, the integration of manipulatives in mathematics instruction influences student learning and enhances academic achievement, as evidenced by gains in test scores. The findings of the present study are consistent with prior intervention-based research investigating the usage of manipulatives and their advantageous impact on mathematical achievement of the students (Fisher, 2023; Keldgord & Ching, 2022; Rich, 2023; Silva et al., 2021; Vallejo-Vargas & Reid, 2024). This convergence across studies reinforces the notion that the broad impact of teaching HCF and LCM through virtual manipulatives extends beyond immediate content mastery, potentially reshaping students' overall disposition toward mathematical abstraction and their capacity to engage with higher-order problems that rely on these foundational principles.

Difference between Classroom Engagement Level – (RQ2)

Another aim of this study was to compare the level of classroom engagement in HCF and LCM concept-oriented class students between both the groups. The findings show that the virtual manipulative group whose intervention was systematically structured was able to increase classroom engagement level of students (Research Question a2). The posttest

scores on the WIHIC scale of the students in the concrete manipulative group ($M = 3.803$, $n = 16$) was significantly low when compared to the posttest scores of students of virtual manipulative group ($M = 4.134$, $n = 16$), with the medium level effect (partial eta-square = 0.466). Increased classroom engagement level within the learning context is not simply a motivational outcome, but more a necessary pedagogical mechanism. According to the sociocultural perspective, the deep conceptual understanding of the relational properties common factors and multiples develops through the active interaction with cultural tools.

When students engage more with others, they are more likely to persist in finding patterns, ask exploratory questions and co-construct meaning, thereby helping to overcome didactic barriers which might otherwise override learning and much of HCF and LCM instruction being simple algorithm transmission. The findings of this study correspond with the experimental studies on teaching mathematics of Furner (2024). When we look at studies across different concepts of mathematics (Guan et al., 2024; Schäfer, 2021), we see that the use of virtual manipulatives positively affects the classroom engagement of students in classrooms. Along with these studies, findings, based on meta-analytics of the benefits of using virtual manipulatives in teaching mathematics (Halim et al., 2017), were in conformity with the empirical findings of this study. The effect size with respect to classroom engagement is medium. When HCF and LCM instruction is specifically the context, we can say that virtual manipulatives clearly keep students engaged, enabling them to visualize, realize, and materialize the abstract number relationships we want them to. This ensures sustained cognitive effort by students when moving from concrete examples to general principles.

In addition, one-way ANCOVA and mixed-design ANOVA were employed to analyze variations in classroom engagement levels on "WIHIC scale" to answer the research question b2. In terms of "WIHIC scale" the findings revealed a statistically significant difference or improvement in mean scores comparing the pretest ($M = 3.803$, $SD = 0.253$) and the posttest ($M = 4.134$, $SD = 0.113$) of virtual manipulative group students. A significant main effect was observed for pre- and post-assessment scores, Wilks' Lambda = 12.423, $F(1, 30) = 488.936$, $p = 0.00$, partial eta squared = 0.942. Both groups exhibited notable improvements in classroom engagement level across the two assessment phases (pre- and post-interventions). The partial eta squared value indicated a large effect size. Following implementation of the treatment, students' mathematical achievement in the virtual manipulative group showed substantial students' engagement in classroom while studying the HCF and LCM mathematical concepts demonstrated significant improvement compared to pre-intervention levels. This remarkable increase in engagement carries profound implications for elementary mathematics education, as active participation during the learning of foundational number theory concepts is directly linked to students' ability to internalize and apply these ideas in diverse mathematical contexts.

When students are cognitively invested in exploring the relationships between multiples and factors through interactive virtual tools, they develop not only procedural fluency but also the conceptual resilience needed to tackle fraction operations, least common denominator problems, and algebraic factorization in subsequent grades—thereby fulfilling the broader curricular goal of building mathematically literate and confident learners. Nevertheless, the findings of study conducted by Halim et al. (2017) confirms, usage of manipulatives in mathematics classroom influences students' engagement in classroom concerning the enhancement in test scores. The results of the present study are consistent with prior intervention-based research examining the usage of manipulatives and their advantageous influence on participation of students and engagement in classroom (Bush, 2021; Kabel et al., 2021; Moyer-Packenham et al., 2008). Collectively, these findings affirm that the strategic deployment of virtual manipulatives in teaching HCF and LCM not only yields superior immediate outcomes but also cultivates an interactive learning ecology that supports students' long-term mathematical development, ensuring that foundational concepts are understood with sufficient depth to serve as durable anchors for future academic growth.

Limitations

This study identifies sample characteristics and teacher effects as the two constraint which might affect generalizability of the findings. Grade five students were the subjects of this study. Only students at the same school were considered with very limited sample size. Limited participants suggest that a change in individual traits can significantly influence comparison results. Furthermore, these students were already familiar with both the concrete and the virtual manipulatives that they had used in class. The findings may not be valid for students of any other age, or students of other ethnic backgrounds, or students of other socioeconomic backgrounds because the study participants were all drawn from the same school that serves mostly a middle-class community and has very little diversity.

Moreover, how well the students learn may depend on the knowledge of the teacher regarding the use of the manipulatives. There may be teacher differences in instructional skill. One may be better than the other at the use of math manipulatives. Thus, the effect could be on the outcome of the study.

Conclusion and Further Research

In conclusion, this study provides robust empirical evidence that virtual manipulatives are significantly more effective than concrete manipulatives in enhancing both mathematical achievement and classroom engagement among fifth-grade students learning HCF and LCM. The virtual manipulative group demonstrated substantially greater posttest gains, with a significant interaction effect indicating that the intervention produced sustained

improvements over time, while the differential engagement outcomes further confirm that virtual manipulatives create more interactive and motivating learning environments. These findings align with the constructivist frameworks underpinning this research, as virtual manipulatives appear to more effectively scaffold students' transition from concrete operational reasoning to abstract theoretical understanding, while simultaneously functioning as cultural tools that foster the social and exploratory dynamics emphasized by sociocultural theory.

Future studies should extend this investigation by analyzing the comparative impacts of concrete and virtual manipulatives over foundational mathematical topics and at different grade levels, thereby accessing the generalizability. Longitudinal designs are particularly warranted to assess whether the achievement gains observed with virtual manipulatives translate into sustained conceptual retention and transfer to higher-order mathematical applications, such as algebraic factorization and fractional operations. Additionally, researchers should explore the potential benefits of blended or sequential approaches that strategically integrate both concrete and virtual manipulatives to capitalize on the tactile advantages of physical objects while leveraging the dynamic customization of digital tools. Investigating the role of individual learner characteristics, such as prior knowledge, cognitive load capacity, and learning preferences, would further refine instructional recommendations and address the concerns regarding potential cognitive overload in virtual environments. Finally, qualitative and mixed-methods inquiries could provide richer insights into the specific interactive features of virtual manipulatives that most powerfully drive student engagement and conceptual understanding, thereby informing the design of more effective digital pedagogical tools for elementary mathematics education.

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